SECTION - A

Maths-1

A.1.

Marks obtained	Frequency	Cumulative frequency
0 - 10	8	8
10 - 20	10	18
20 - 30	12	30
30 - 40	22	52
40 - 50	30	82
50 - 60	18	100

Here Total no. of observations = 100 = n.

$\frac{n}{2} = 50.$

$\therefore$ Median class = $\boxed{30 - 40}$

$\boxed{A.2}$ Total no. of balls = No. of red balls + No. of black balls

$= 4 + 6$

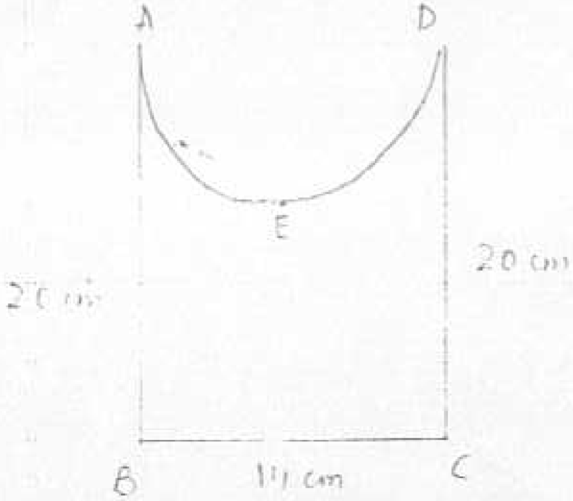
$= 10$

No. of black balls $= 6$.

$$P(\text{black ball}) = \frac{\text{No. of black balls}}{\text{Total no. of balls}} = \frac{6}{10} = \frac{3}{5}$$

$$P(\text{black ball}) = \boxed{\frac{3}{5}}$$

$\boxed{A.3}$



~~For~~ For rectangle ABCD,

length $= L = 20$ cm

breadth $= b = 14$ cm

For semicircle $\overset{\frown}{AED}$,

~~radi~~ diameter = breadth of rectangle

$= d = 14$ cm.

Perimeter of figure = AB + BC + CD + length of arc $\overset{\frown}{AED}$ M-3

$= 20 + 14 + 20 + (\pi r)$

$= 54 + \frac{22}{7} \times 7$

$= 76$ cm

Perimeter = $\boxed{76 \text{ cm}}$

$\boxed{A.4}$

$\sin 3\theta = \cos(\theta - 6)^\circ$

$\Rightarrow \cos(90^\circ - 3\theta) = \cos(\theta - 6^\circ)$ $\left[\because \sin\theta = \cos\theta(90^\circ - \theta)\right]$

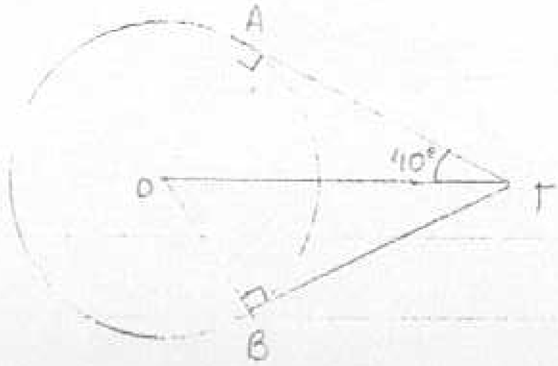
$\Rightarrow 90^\circ - 3\theta = \theta - 6^\circ$

$\Rightarrow 4\theta = 96^\circ$

$\Rightarrow \theta = 24^\circ$

$\therefore \boxed{\theta = 24^\circ}$

A·5



Given : In $c(O, OA)$, $\angle ATO = 40^\circ$.

To find : $\angle AOB$.

Solution : AT and BT are tangents to $c(O, OA)$.

$\Rightarrow AT = BT$ [Tangents from an external point are equal] —(1).

In Δs AOT and BOT

$AT = BT$ [From (1)].

$AO = OB$ [radii].

$OT = TO$ [common]

$\therefore \Delta AOT \cong \Delta BOT$ [SSS congruency].

$\Rightarrow \angle AOT = \angle BOT$ and [cpct] —(2)

$\angle ATO = \angle BTO$ —(3)

$\Rightarrow \angle BTO = \angle ATO = 40^\circ$ [from (3)].

In ΔBTO,

$\angle OBT = 90^\circ$ [$\because$ Tangent is $\perp$r to radius through point of contact]. —(4).

~~$\angle AO$~~ $\angle BOT + \angle OTB + \angle OBT = 180^\circ$ [Angle Sum Property of a Δ].

$\Rightarrow \angle BOT + 40^\circ + 90^\circ = 180^\circ$ [From (3),(4)].

$\Rightarrow \angle BOT = 180^\circ - 130^\circ$

$\Rightarrow \angle BOT = 50^\circ$.

$\angle AOT = \angle BOT = 50^\circ$ [From (2)] — (5).

$\angle AOB = \angle AOT + \angle BOT$

$= 50^\circ + 50^\circ$ [from (5)].

$= 100^\circ$

$\therefore \boxed{\angle AOB = 100^\circ}$

A.6 $\sqrt{2} = 1.414\ldots$ (approx)

$\sqrt{3} = 1.732\ldots$ (approx).

$\therefore$ Rational no. between $\sqrt{2}$ and $\sqrt{3}$ is $1.6 = \frac{16}{10} = \frac{8}{5}$.

Required rational no. between $\sqrt{2}$ and $\sqrt{3}$ is $\boxed{\frac{8}{5} \text{ or } 1.6}$.

A.7 Since the graph of $y = f(x)$ intersects the x-axis at 3 points,

No. of zeroes of polynomial $y = f(x)$ is $\boxed{3}$.

A.8 $2x^2 + 5x - 12 = 0$.

L.H.S. : $2(-4)^2 + 5(-4) - 12$.

$= 2(16) + (-20) - 12$.

$= 32 - 20 - 12$

$= 32 - 32$

$= 0$.

R.H.S. $= 0$.

Since LHS = RHS = 0.

$\Rightarrow$ $x = -4$ satisfies the equation $2x^2 + 5x - 12 = 0$.

$\therefore$ $\boxed{x = -4 \text{ is a solution}}$ of the equation $2x^2 + 5x - 12 = 0$.

$\boxed{\text{A 9}}$ AP is $\sqrt{8}$, $\sqrt{18}$, $\sqrt{32}$ ...

This is the same as $2\sqrt{2}$, $3\sqrt{2}$, $4\sqrt{2}$ ...

Here

first term $= a_1 = 2\sqrt{2}$.

~~common~~ common difference $= a_2 - a_1 = d = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$.

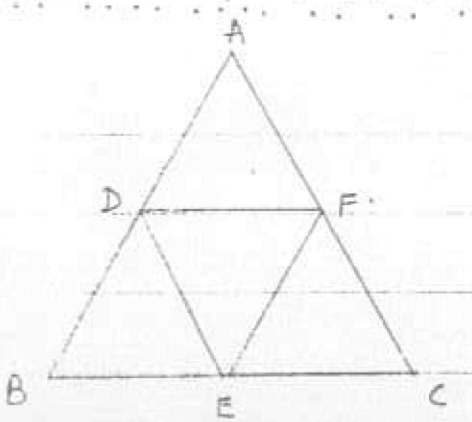
a_2 = second term $= 3\sqrt{2}$.

a_3 = third term $= 4\sqrt{2}$.

a_4 = fourth term $= a_3 + d = 4\sqrt{2} + \sqrt{2} = 5\sqrt{2} = \sqrt{50}$.

$\therefore$ Next term of the AP is $\boxed{\sqrt{50}}$.

A.10



Given : In ΔABC, D, F and E are the mid-points of AB, AC and BC respectively.

To find : $\dfrac{ar\,\Delta DEF}{ar\,\Delta ABC}$.

Solution : Since D and F are mid-points of AB and AC,

$\Rightarrow DF = 2BC$. [Mid-point theorem].

$\Rightarrow \dfrac{DF}{BC} = \dfrac{1}{2}$ — (1)

Similarly, $\dfrac{EF}{AB} = \dfrac{1}{2}$ —(2)

Also, $\dfrac{DE}{AC} = \dfrac{1}{2}$ —(3).

From (1), (2), (3),

In Δs DEF and ABC.

$$\frac{DF}{BC} = \frac{EF}{AB} = \frac{DE}{AC} = \frac{1}{2}$$

$\therefore \Delta DEF \sim \Delta CAB$ [SSS similarity].

$$\frac{ar\,\Delta DEF}{ar\,\Delta CAB} = \left(\frac{DF}{BC}\right)^2$$

[~~Areas of~~ Ratios of areas of two similar Δs is equal to the square of the ratio of their corresponding sides].

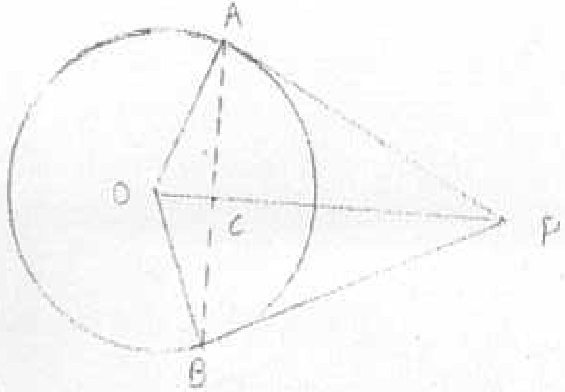
$$\Rightarrow \frac{ar\,\Delta DEF}{ar\,\Delta CAB} = \left(\frac{1}{2}\right)^2$$

[From (1)].

$$\therefore \boxed{\frac{ar\,\Delta DEF}{ar\,\Delta ABC} = \frac{1}{4}}$$

SECTION - 'B'

A·11



Given : In $c(O, OA)$. OP = diameter of circle.

To prove : $\triangle ABP$ is equilateral.

Construction : Join AB intersecting OP at C.

Proof: Here, OP = diameter of $c(O, OA)$.

$\Rightarrow OP = 2 \times$ radius.

$\Rightarrow OP = 2OA$.

$\Rightarrow \dfrac{OP}{OA} = \dfrac{2}{1}$ — (1).

Also, $\angle OAP = \angle OBP = 90^\circ$ [Given].

$\Rightarrow$ [illegible] to circle [A line ⊥ to radius through [illegible]

In right $\triangle OAP$,

$\frac{OP}{AO} = \frac{2}{1}$ [from (1)].

$\Rightarrow \frac{AO}{OP} = \frac{1}{2}$.

$\Rightarrow \sin P = \frac{1}{2}$.

$\Rightarrow \sin P = \sin 30^\circ$

$\Rightarrow P = 30^\circ$.

$\Rightarrow \angle APO = 30^\circ$ —(2).

In $\triangle s$ AOP and BOP,

$BO = AO$. [radii].

$\angle OAP = \angle OBP = 90^\circ$. [given].

$OP = PO$. [common]

$\therefore \triangle AOP \cong \triangle BOP$ [RHS congruency].

$\Rightarrow \angle AOP = \angle BOP$ and [cpct]. -(3)

$\Rightarrow \angle BPO = \angle APO = 30^\circ$ [From (2)] . — (4)

Also, In $\triangle AOP$,

$\angle AOP + \angle OPA + \angle PAO = 180^\circ$ [Angle sum property of a $\triangle$] .

$\Rightarrow \angle AOP + 30^\circ + 90^\circ = 180^\circ$ [From (4)] .

$\Rightarrow \angle AOP = 180^\circ - 120^\circ$

$\Rightarrow \angle AOP = 60^\circ$. — (5)

In $\triangle$s AOC and BOC,

$\angle AOC = \angle BOC$. [From (3)]

$AO = BO$. [radii] .

$OC = CO$ [common] .

$\therefore \triangle AOC \cong \triangle BOC$ [SAS congruency] .

$\Rightarrow \angle OAC = \angle OBC$. [cpct] — (6)

$\angle OCA = \angle OCB$. — (7).

$\angle ACO + \angle OCB = 180^\circ$ [linear pair] .

$\Rightarrow 2\angle ACO = 180^\circ$ [From (7)]

$\Rightarrow \angle ACO = 90^\circ$. — (8)

In ΔAOC,

$\angle AOC + \angle ACO + \angle CAO = 180^\circ$ [Angle sum property of a Δ].

$\Rightarrow 60^\circ + 90^\circ + \angle CAO = 180^\circ$ [From (5), (8)].

$\Rightarrow \angle CAO = 180^\circ - 150^\circ$

$\Rightarrow \angle OAC = 30^\circ$. —(9).

Also, $\angle OAP = 90^\circ$ [Given].

$\Rightarrow \angle OAC + \angle CAP = 90^\circ$

$\Rightarrow 30^\circ + \angle CAP = 90^\circ$

$\Rightarrow \angle CAP = 60^\circ$. —(10).

Since $\angle OAC = 30^\circ$

$\Rightarrow \angle OBC = \angle OAC = 30^\circ$ [From (6)].

$\Rightarrow \angle PBC = 60^\circ$ —(11).

$\angle APB = \angle APO + \angle BPO$

$= 30^\circ + 30^\circ$ [From (4)].

$= 60^\circ$ ~~[From~~

$\Rightarrow \angle APB = 60^\circ$ —(12).

From (10), (11) and (12),

In Δ APB,

$\angle PBA = \angle CAP = \angle APB = 60^\circ$.

$\therefore \Delta APB$ is an equilateral Δ [$\because$ All angles are 60° each].

Hence proved.

$\boxed{A.12.}$ $ax^2 - 6x - 6 = p(x)$.

Product of zeroes = 4.

Also,

Product of zeroes = $\dfrac{\text{Constant term}}{\text{Coefficient of } x^2}$

$\Rightarrow 4 = \dfrac{-6}{a}$

$\Rightarrow a = -\dfrac{6}{4}$

$\Rightarrow a = -3$ [$\therefore a = -3$]

A·13 For what value of k are the points . . .

A (1, 1) $x_1 = 1$ $y_1 = 1$

B (3, k) $x_2 = 3$ $y_2 = k$

C (−1, 4). $x_3 = -1$ $y_3 = 4$.

since ~~If~~ A, B and C are collinear.

$\Rightarrow$ Ar ABC = 0.

$\Rightarrow \frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)] = 0$.

$\Rightarrow \frac{1}{2}[1(k-4) + 3(4-1) + (-1)(1-k)] = 0$.

$\Rightarrow k - 4 + 9 \cancel{+} -1 + k = 0$.

$\Rightarrow 2k + 9 - 5 = 0$.

$\Rightarrow 2k + 4 = 0$

$\Rightarrow k = -\frac{4}{2}$

$\therefore \boxed{k = -2}$.

A.14

Total no. of cards = 50 − 5 + 1 = 46.

(i) No. of cards with a prime no less than 10
= 2 i.e. 5 and 7.

P (prime no less than 10) = $\frac{\text{No. of cards with a prime no. less than 10}}{\text{Total no. of cards}}$

$= \frac{2}{46} = \frac{1}{23}$

(ii) No. of cards with a perfect square no. = 5 (i.e. 9, 16, 25, 36, 49).

P(perfect square no.) = $\frac{\text{No. of cards with a perfect square no.}}{\text{Total no. of cards}}$

$= \frac{5}{46}$.

(i) P (prime no. less than 10) = $\boxed{\frac{1}{23}}$

(ii) P (perfect square no.) = $\boxed{\frac{5}{46}}$

$\boxed{A \cdot 15}$ $7\sin^2\theta + 3\cos^2\theta = 4$

$\Rightarrow 7\sin^2\theta + 3(1-\sin^2\theta) = 4$ $[\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow \cos^2\theta = 1-\sin^2\theta]$.

$\Rightarrow 7\sin^2\theta + 3 - 3\sin^2\theta = 4$.

$\Rightarrow 4\sin^2\theta = 1$.

$\Rightarrow \sin^2\theta = \frac{1}{4}$

$\Rightarrow \sin\theta = \sqrt{\frac{1}{4}}$

$\Rightarrow \sin\theta = \frac{1}{2}$. — (1)

$\cos\theta = \sqrt{1-\sin^2\theta}$ $[\because \sin^2\theta + \cos^2\theta = 1]$.

$= \sqrt{1-\left(\frac{1}{2}\right)^2} = \sqrt{1-\frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$. — (2).

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$ [From (1), (2)].

$\boxed{\tan\theta = \frac{1}{\sqrt{3}}}$ Hence proved.

SECTION - 'C'

A-16

~~By Euclid's division lem~~

Let a be any positive integer. Then,

By Euclid's division lemma,

i.e. $a = bq + r \quad 0 \leq r < b$.

a can be of the form $3q$, $3q+1$ or $3q+2$, where q is some in

Case I: $a = 3q$.

$a^2 = (3q)^2 = 9q^2 = 3(3q^2) = 3m$ where $m = 3q^2$ —(1).

Case II: $a = 3q+1$.

$a^2 = (3q+1)^2 = 9q^2 + 6q + 1 \quad [\because (a+b)^2 = a^2 + 2ab + b^2]$.

$= 3(3q^2 + 2q) + 1$

$= 3m + 1$ where $m = 3q^2 + 2q$. —(2).

Case III: $a = 3q+2$.

$a^2 = (3q+2)^2 = 9q^2 + 12q + 4 \quad [\because (a+b)^2 = a^2 + 2ab + b^2]$.

$= 3(3q^2 + 4q + 1) + 1$.

$= 3m + 1$ where $m = 3q^2 + 4q + 1$. — (3).

$\therefore$ From (1), (2) and (3),

we conclude,

The square of any positive integer is of the form $3m$ or $3m+1$ for some integer m.

Hence proved.

$\boxed{A.17}$

$37x + 43y = 123$ — (1)

$43x + 37y = 117$ — (2)

Adding (1) and (2),

$$\begin{array}{r} 37x + 43y = 123 \\ 43x + 37y = 117 \\ \hline 80x + 80y = 240 \\ \hline \end{array}$$

$\Rightarrow 80(x+y) = 80(3)$.

$\Rightarrow x + y = 3$ — (3).

~~[illegible]~~

Subtracting (2) from (1),

$$37x + 43y = 123$$

$$43x + 37y = 117$$

$$(-) \quad (-) \quad (-)$$

$$-6x + 6y = 6.$$

$$\Rightarrow -6(x-y) = -6(-1)$$

$$\Rightarrow x - y = -1. \quad -(4)$$

Adding (3) and (4),

$$x + y = 3$$

$$x - y = -1.$$

$$2x = 2.$$

$$\Rightarrow x = 1.$$

Substituting $x = 1$ in (4),

$$1 - y = -1.$$

$$\Rightarrow -y = -2.$$

$$\Rightarrow y = 2.$$

$$\therefore \boxed{x = 1}, \boxed{y = 2}$$

A18 Let us assume, to the contrary that $\sqrt{5}$ is rational. Then,
$\sqrt{5} = \frac{a}{b}$, where a and b are positive coprime integers and $b \neq 0$

$\Rightarrow a = \sqrt{5}b$.

$\Rightarrow a^2 = 5b^2$ [Squaring both sides]. —(1)

$\Rightarrow$ 5 divides $5b^2$

$\Rightarrow$ 5 divides a^2 [$\because a^2 = 5b^2$].

$\Rightarrow$ 5 divides a [$\because$ If p divides a^2 then p divides a] —(2)

$\Rightarrow a = 5c$ for some integer c.

$\Rightarrow a^2 = 25c^2$ [Squaring both sides].

$\Rightarrow 5b^2 = 25c^2$ [$\because a^2 = 5b^2$ [From (1)]].

$\Rightarrow b^2 = 5c^2$ —(3)

$\Rightarrow$ 5 divides $5c^2$

$\Rightarrow$ 5 divides b^2 [From (3), $b^2 = 5c^2$].

$\Rightarrow$ 5 divides b [$\because$ If p divides a^2, then p divides a] —(4)

From (2) and (4),
5 is a common factor of a and b

But this contradicts ~~the fact~~ that a and b are coprime
i.e. they ~~have~~ no common factor apart from 1.
This means our assumption is wrong.
$\sqrt{5}$ is an irrational no. Hence proved.

$\boxed{A.19}$ Let the AP be $a_1, a_2, a_3, a_4 \ldots$ where
first term $= a$
common difference $= d$.
Then,

$a_n = a + (n-1)d$.

$\Rightarrow a_4 = a + (4-1)d$

$\Rightarrow a_4 = a + 3d$. $\quad -(1)$

$a_8 = a + (8-1)d$

$\Rightarrow a_8 = a + 7d$. $\quad -(2)$

$a_6 = a + (6-1)d$

$\Rightarrow a_6 = a + 5d$. $\quad -(3)$.

$a_{10} = a + (10-1)d$

Also,

$a_4 + a_8 = 24.$

$\Rightarrow a+3d+a+7d = 24$ [From (1), (2)].

$\Rightarrow 2a+10d = 24.$

$\Rightarrow a+5d = 12.$ --(5)

$a_6 + a_{10} = 44.$

$\Rightarrow a+5d+a+9d = 44$ [from (3), (4)].

$\Rightarrow 2a+14d = 44$

$\Rightarrow a+7d = 22$ --(6).

Subtracting (5) from (6),

$$\begin{array}{r} \cancel{a}+7d = 22 \\ \cancel{a}+5d = 12. \\ (-)\quad(-)\quad\;(-) \\ \hline 2d = 10. \end{array}$$

$\Rightarrow d = 5$

Substituting $d = 5$ in (5),

$a + 5(5) = 12.$

$\Rightarrow a + 25 = 12.$

$\Rightarrow a = -13.$

$a_1 = -13$

$a_2 = a_1 + d = -13 + 5 = -8.$

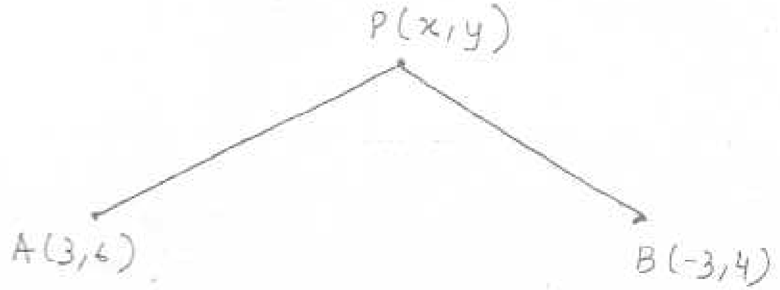
$a_3 = a_2 + d = -8 + 5 = -3.$

First three terms of the AP are $\boxed{-13, -8 \text{ and } -3}$.

~~A-Q0~~

CBSE

A 20



Here.

A (3, 6) $\quad x_1 = 3 \quad y_1 = 6$

B (-3, 4) $\quad x_2 = -3 \quad y_2 = 4.$

P (x, y) ;

According to Problem,

P is equidistant from A and B.

$\Rightarrow AP = BP$

$\Rightarrow AP^2 = PB^2$

By distance formula.

distance between two points = $\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

Now,

$AP^2 = PB^2$

$\Rightarrow (x_1 - x)^2 + (y_1 - y)^2 = (x_2 - x)^2 + (y_2 - y)^2$ [Distance formula].

$\Rightarrow (3 - x)^2 + (6 - y)^2 = (-3 - x)^2 + (4 - y)^2$

$\Rightarrow \cancel{9} + \cancel{x^2} - \underline{6x} + \underline{36} + \cancel{y^2} - \underline{12y} = \cancel{9} + \cancel{x^2} + \underline{6x} + \underline{16} + \cancel{y^2} - \underline{8y}$

$\Rightarrow -12y + 8y - 6x - 6x + 36 - 16 = 0.$

$\Rightarrow -4y - 12x + 20 = 0.$

$\Rightarrow -12x - 4y + 20 = 0.$

$\Rightarrow -4(3x + y - 5) = -4(0)$

$\Rightarrow 3x + y - 5 = 0.$

$\boxed{3x + y - 5 = 0}.$

Hence proved.

A·21 To prove: $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta.$

LHS $= (\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2$

$= \sin^2\theta + \operatorname{cosec}^2\theta + 2\sin\theta\operatorname{cosec}\theta + \cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta$

$[\because (a+b)^2 = a^2 + 2ab + b^2].$

$= \sin^2\theta + \cos^2\theta + \operatorname{cosec}^2\theta + \sec^2\theta + 2\cancel{\sin\theta}\left(\frac{1}{\cancel{\sin\theta}}\right) + 2\cancel{\cos\theta}\left(\frac{1}{\cancel{\cos\theta}}\right)$

$\left[\because \operatorname{cosec}\theta = \frac{1}{\sin\theta},\ \sec\theta = \frac{1}{\cos\theta}\right].$

$= 1 + \operatorname{cosec}^2\theta + \sec^2\theta + 2 + 2.$ $\quad [\because \sin^2\theta + \cos^2\theta = 1$

$= 5 + \operatorname{cosec}^2\theta + \sec^2\theta$

$= 5 + 1 + \cot^2\theta + 1 + \cancel{\cos^2\theta}\ \tan^2\theta$ $\quad [\because \operatorname{cosec}^2\theta - \cot^2\theta = 1.$

$\sec^2\theta - \tan^2\theta = 1].$

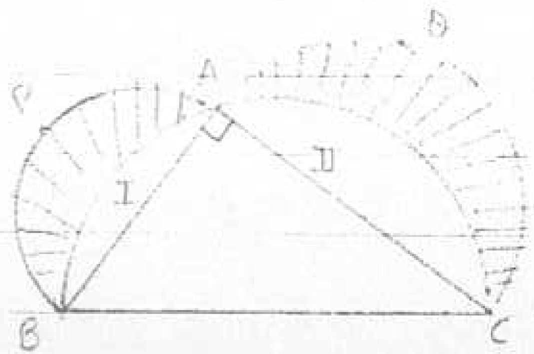
$= 7 + \tan^2\theta + \cot^2\theta$

$= R.H.S.$

Since LHS = RHS.

Hence verified.

A.22.



In ΔABC, right angled at A,

$AB^2 + AC^2 = BC^2$ [Pythagoras Theorem].

$\Rightarrow 3^2 + 4^2 = BC^2$

$\Rightarrow BC^2 = 9 + 16$

$\Rightarrow BC^2 = 25$.

$\Rightarrow BC = 5$ ~~cm~~ units

$\Rightarrow$ Diameter of semicircle $\overset{\frown}{BAC} = BC = 5$ units $= d$.

radius $= r = \frac{d}{2} = \frac{5}{2}$ ~~cm~~ units.

Area of semicircle $\overset{\frown}{BAC} = \frac{\pi r^2}{2} = \left(\frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times \frac{1}{2}\right)$ ~~cm~~ units^2 —(1)

Diameter of semicircle $\overset{\frown}{APB}$ = $AB = 3$ ~~cm~~ units $= d_1$.

radius $= r_1 = \frac{d_1}{2} = \frac{3}{2}$ ~~cm~~ units.

Area of semicircle $\overset{\frown}{APB} = \frac{\pi r_1^2}{2} = \left(\frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times \frac{1}{2}\right)$ ~~cm²~~ units² —(2

Diameter of semicircle AQC = $AC = 4$ ~~cm~~ units $= d_2$.

radius $= r_2 = \frac{d}{2} = \frac{4}{2} = 2$ ~~cm~~ units.

Area of semicircle $\overset{\frown}{AQC} = \frac{\pi r^2}{2} = \left(\frac{22}{7} \times 2 \times 2 \times \frac{1}{2}\right)$ ~~cm²~~ units² — (3

~~Area of shaded region~~

Area of $\Delta ABC = \frac{1}{2} \times$ base $\times$ height $= \frac{1}{\cancel{2}} \times \overset{2}{\cancel{4}} \times 3 = 6$ ~~cm²~~ units². —(1

~~Area of shaded region = Area of semicircle APB + Area of semicircle~~
~~− (Area of semicircle BAC − Area of ΔABC).~~

~~$= \frac{1}{2} \times \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} + \frac{1}{2} \times \frac{22}{7} \times 2 \times 2 \times \frac{1}{2} - \left(\frac{1}{2} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2}\right) - 6)$~~

$= \left(\frac{1}{2} \times \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} + \frac{1}{2} \times \frac{22}{7} \times 2 \times 2 \times \frac{1}{2} - \frac{1}{2} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2}\right) + 6.$

$= \frac{1}{2} \times \frac{22}{7} \times \left(\frac{3}{2} \times \frac{3}{2} + 2 \times 2 - \frac{5}{2} \times \frac{5}{2}\right) + 6.$

$= \frac{11}{7}\left(\frac{9}{4} + \frac{16}{4} - \frac{25}{4}\right.$

Area of I + II = Area of semicircle $\widehat{BAC}$ − Area of $\triangle ABC$. [From (1) & (4)].

$= \frac{1}{2} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} - 6.$

$= \frac{275}{28} - \frac{168}{28}$

$= \frac{107}{28}$ units^2 — (5)

Area of shaded region = Area of semicircle APB + Area of semicircle $\widehat{AQC}$ − Area of I + II.

$= \frac{1}{2} \times \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} + \frac{1}{2} \times \frac{22}{7} \times \frac{4}{2} \times \frac{4}{2} - \left(\frac{107}{28}\right)$ [From (2), (3), (5)].

$$= \frac{1}{\cancel{2}} \times \frac{\cancel{22}^{11}}{7} \times \frac{1}{2} \times \frac{1}{2}(9+16) - \frac{107}{28}$$

$$= \frac{11 \times 25}{28} - \frac{107}{28}$$

$$= \frac{275 - 107}{28}$$

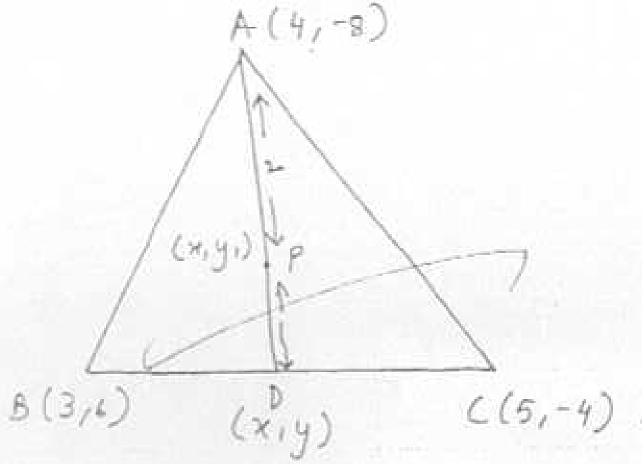
$$= \frac{168}{28}$$

$$= \frac{24}{4}$$

$$= 6 \text{ units}^2$$

Area of shaded region is $\boxed{6 \text{ sq. units}}$

A. 23



Here in $\triangle ABC$.

A (4, -8)

B (3, 6)

C (5, -4).

D is mid-point of BC.

By mid-pt. formula, $x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$.

$\Rightarrow x = \frac{3+5}{2}$, $y = \frac{6-4}{2}$.

$\Rightarrow x = 8$, $y = 2$

∴ Coordinates of D are D (4, 1).

Now,

$\frac{AP}{PD} = \frac{2}{1}$ Let AP = m and PD = n.

$\underset{A}{(4,-8)} \xleftarrow{\ 2\ } \xrightarrow{} \underset{P}{(x_1, y_1)} \xleftarrow{\ 1\ }$

By section formula, $x = \frac{mx_2 + nx_1}{m+n}$, $y = \frac{my_2 + ny_1}{m+n}$.

$\Rightarrow x_1 = \frac{2(4) + 1(4)}{2+1}$, $y_1 = \frac{2(1) + 1(-8)}{2+1}$

$\Rightarrow x_1 = \frac{8+4}{3}$, $y_1 = \frac{2-8}{3}$.

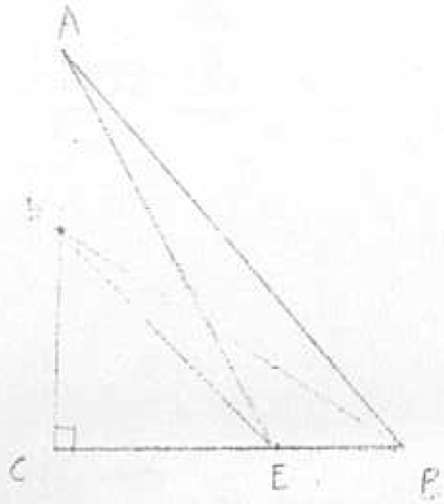
$\Rightarrow x_1 = \frac{12}{3}$, $y_1 = \frac{-6}{3}$.

$\Rightarrow x_1 = 4$, $y_1 = -2$.

∴ Coordinates of P are $\boxed{P(4, -2)}$.

(4.1)

$\boxed{A \cdot 24}$



Given : In ΔACB, $\angle ACB = 90^\circ$, D and E are points on AC and BC respectively.

To prove : $AE^2 + BD^2 = AB^2 + DE^2$.

Construction : Join DE, AE and BD.

Proof : By Pythagoras Theorem,

In ΔACB,

$AC^2 + CB^2 = AB^2$ — (1)

In ΔDCE,

$DC^2 + CE^2 = DE^2$ — (2)

In ΔDCB,

$DC^2 + CB^2 = DB^2$ —(3)

In ΔACE,

$AC^2 + CE^2 = AE^2$ —(4).

Adding (1) and (2),

$AB^2 + DE^2 = AC^2 + BC^2 + DC^2 + CE^2$

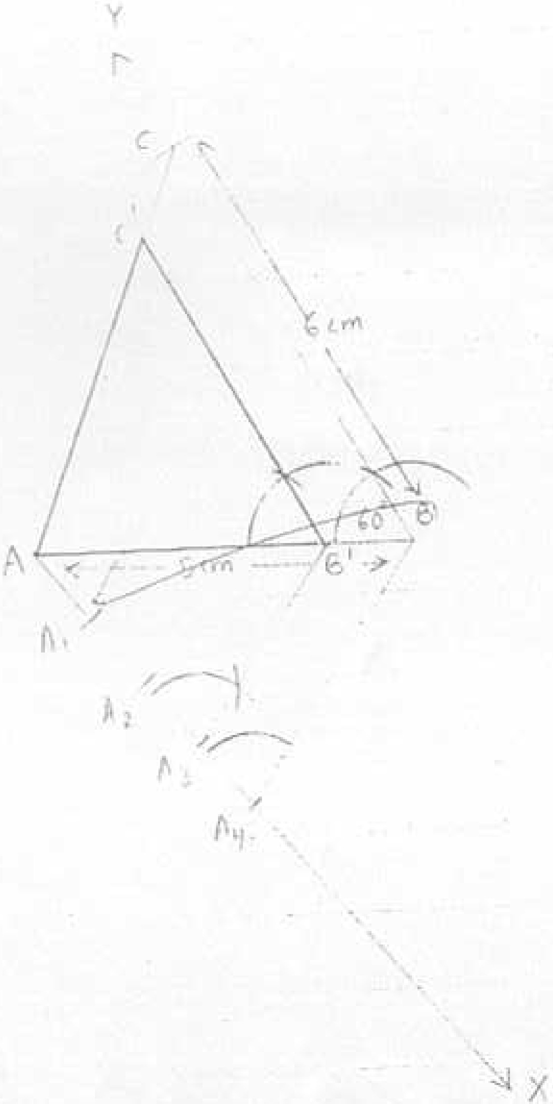
$= (AC^2 + CE^2) + (BC^2 + DC^2)$ [Rearranging terms].

$= AE^2 + BD^2$ [From (3), (4)].

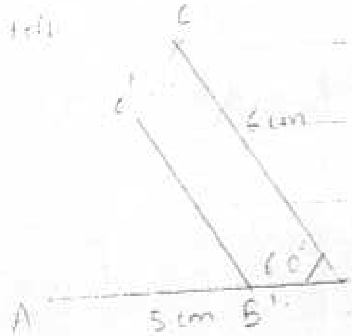
$\therefore \boxed{AB^2 + DE^2 = AE^2 + BD^2}$

Hence proved

A 25.



rough sketch



$\therefore \triangle AB'C'$ is the required $\triangle$

$$\frac{AB'}{AB} = \frac{B'C'}{BC} = \frac{AC'}{AC} = \frac{3}{4}.$$

$\triangle ABC \sim \triangle AB'C'$.

In $\triangle ABC$,

$AB = 5$ cm

~~BC = 6 cm~~

$\angle ABC = 60^\circ$.

SECTION-'D'

M-37

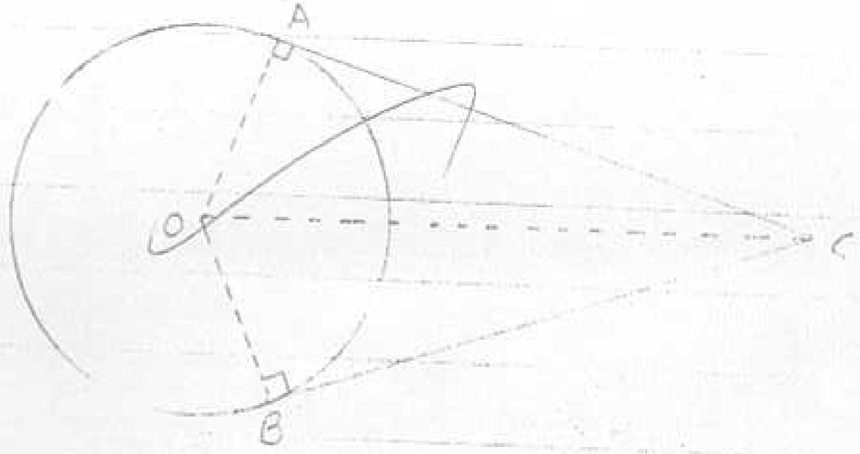
A.26.

Given : In $c(O, OA)$, AC and BC are tangents to the circle from point C.

To prove : AC = BC.

Construction : Join OA, OB and OC.

Figure :



Proof : In Δs AOC and BOC

AO = OB [radii of same circle].

OC = CO [common].

$\angle OAC = \angle OBC = 90°$ [$\because$ Tangent is $\perp$r to radius through point of contact].

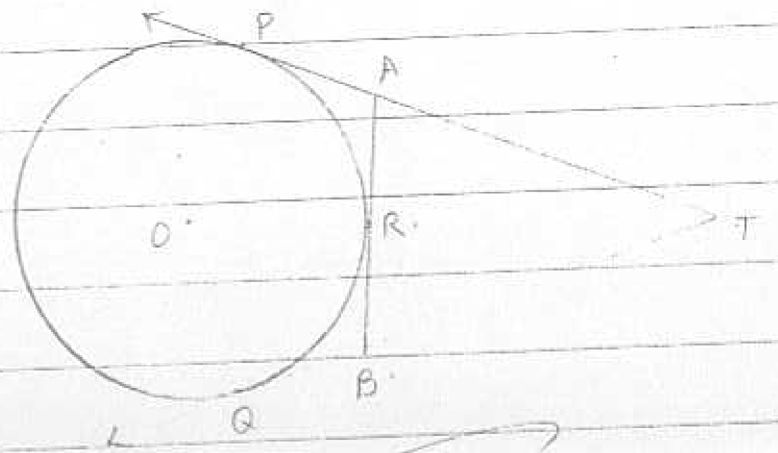
$\Rightarrow \Delta AOC \cong \Delta BOC$ [RHS congruency].

$\Rightarrow$ AC = BC [cpct].

$\therefore$ AC = BC

Hence proved.

Rider:



Given : In $c(O, OP)$, PT ~~and~~ QT are tangents from PT to circle.

R is a point on the circle, AB is a tangent to the circle at R.

To prove : $TA + AR = TB + BR$.

Proof : Since tangents from an external point are equal

$\Rightarrow PT = QT$ — (1)

$AP = AR$. — (2)

$BR = BQ$. — (3)

Now,

$PT = QT$ [from (1)]

$\Rightarrow PA + AT = QB + BT$

$\Rightarrow AR + AT = BR + BT$ [~~from~~ (2), (3)]

$TA + AR = TB + BR$

A.27 Let the time taken by smaller pipe to fill the tank separately be x hrs.
Then time taken by larger pipe to fill tank separately $= (x-10)$ hrs.

Part of tank filled by small pipe in x hrs $= 1$.

" " " " " " " " 1 hr $= \frac{1}{x}$

" " " " " " " " $9\frac{3}{8}$ hr $= 9\frac{3}{8} \cdot \frac{1}{x} = \frac{75}{8x}$ — (1)

Part of tank filled by large pipe in ~~hr~~ $(x-10)$ hrs $= 1$.

" " " " " " " 1 hr $= \frac{1}{x-10}$.

" " " " " " " " $9\frac{3}{8}$ hrs $= \frac{75}{8(x-10)}$ — (2).

A to Q,

$$\frac{75}{8x} + \frac{75}{8(x-10)} = 1.$$

$$\Rightarrow \frac{75}{8}\left(\frac{1}{x} + \frac{1}{x-10}\right) = 1$$

$\Rightarrow \frac{1}{x} + \frac{1}{x-10} = \frac{8}{75}$

$\Rightarrow \frac{x-10+x}{8x(x+10)} = \frac{8}{75}$

$\Rightarrow \frac{2x-10}{x^2+10x} = \frac{8}{75}$

$\Rightarrow 150x - 750 = 8x^2 - 10x$

$\Rightarrow 8x^2 + 10x - 150x + 750 = 0$

$\Rightarrow 8x^2 - 140x + 750 = 0$

$\Rightarrow 4x^2 - 70x + 375 = 0$

$\Rightarrow$

$\Rightarrow \frac{x-10+x}{x(x-10)} = \frac{8}{75}$

$\Rightarrow \frac{2x-10}{x^2-10x} = \frac{8}{75}$

$\Rightarrow 150x - 750 = 8x^2 - 80x$

$\Rightarrow 8x^2 - 230x + 750 = 0$

अपना अनुक्रमांक इस उत्तर-पुस्तिका

$\Rightarrow 4x^2 - 11...$

$\Rightarrow 4x^2 - 100x - 15x + 375 = 0.$

$\Rightarrow 4x(x-25) - 15(x-25) = 0.$

$\Rightarrow (4x-15)(x-25) = 0.$

$\Rightarrow (4x-15) = 0$ or $x - 25 = 0.$

$\Rightarrow x = \frac{15}{4}$ or $x = 25.$

When $x = \frac{15}{4}$,

$x - 10 = \frac{15}{4} - \frac{40}{4} = \cancel{-25} \; \frac{-25}{4}$

Time cannot be -ve

$3 \overline{375}$

$2 | 1500$
$2 | 750$
$3 | 375$
$5 | 125$
$5 | 25$
$5 | 5$
$\;\; | 1$

750
125
375

$\Rightarrow x = \frac{15}{4}$ is not possible.

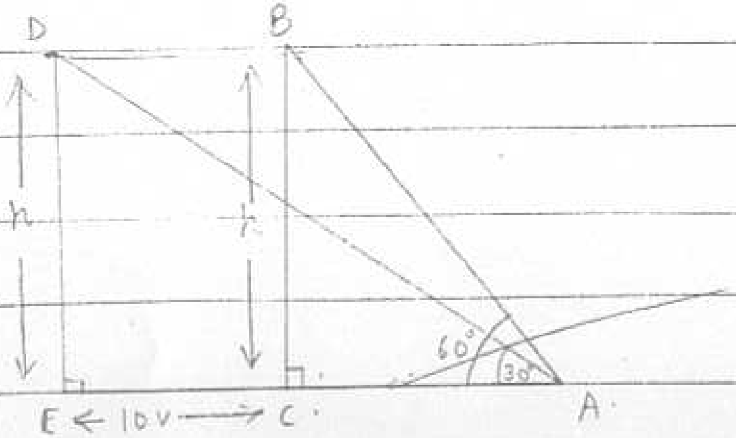
$\Rightarrow x = 25$ hrs.

$x - 10 = 25 - 10 = 15$ hrs

~~∴ Time taken by small pipe is 25 hrs and that taken by larger pipe is 25 hrs.~~

∴ Time taken by small pipe is $\boxed{25 \text{ hrs}}$ and time taken by larger pipe is $\boxed{15 \text{ hrs}}$

A.28



Let the jet originally be at B and let C be the ground. Then,

A is the point of observation

$\Rightarrow \angle BAC = 60^\circ$.

Let the new position of jet be D. Then,

$\angle DAE = 30^\circ$.

Let the height at which the jet is flying be h m above ground.

Let speed of the jet be v m/s. Then,

$t = 10s$.

distance $= \cancel{CD} = 10v\ \cancel{s}\ m = CE$ — (1)

sla

In ΔBAC, right angled at C,

$$\tan 60^\circ = \frac{BC}{AC}$$

$$\Rightarrow \sqrt{3} = \frac{h}{AC}$$

$$\Rightarrow h = \sqrt{3}\,AC \qquad -(2)$$

For

di

In ΔDAE, right angled at E,

$$\tan 30^\circ = \frac{DE}{AE}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{AC+CE}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{\sqrt{3}\,AC}{AC+10v} \qquad [\text{From } (1) \,\&\, (2)]$$

$$\Rightarrow AC + 10v = 3AC$$

$\Rightarrow 2AC = 10v$

$\Rightarrow AC = 5v.$

Substituting $AC = 5v$ in (1),

$h = \sqrt{3}(5v).$

$= 5\sqrt{3}v.$ — (3)

~~speed = 648 km/hr.~~

$\cancel{= \left(\frac{648}{1000} \div \frac{1}{3600}\right) m/s}$

$\cancel{= \frac{640}{1000} \times 3600 \ m/s} = \cancel{\frac{648}{5 \ 1000} \times \frac{18}{3600}}$

$\cancel{= 2304 \ m/s}$

=

640

64
36
384

648
18
5184
480
664

Speed = 648 km/hr

$\Rightarrow v = \dfrac{6480\cancel{00}}{36\cancel{00}}$ m/s.

$\Rightarrow v = \dfrac{1080}{6}$

$\Rightarrow v = 180$ m/s.

Substituting $v = 180$ m/s in

$h = 5\sqrt{3}\, v$

$= 5\sqrt{3} \times 180$

$= 900\sqrt{3}$

$= 900 \times 1.732$

$= 1558.80$ m. $= 1.5588$ km

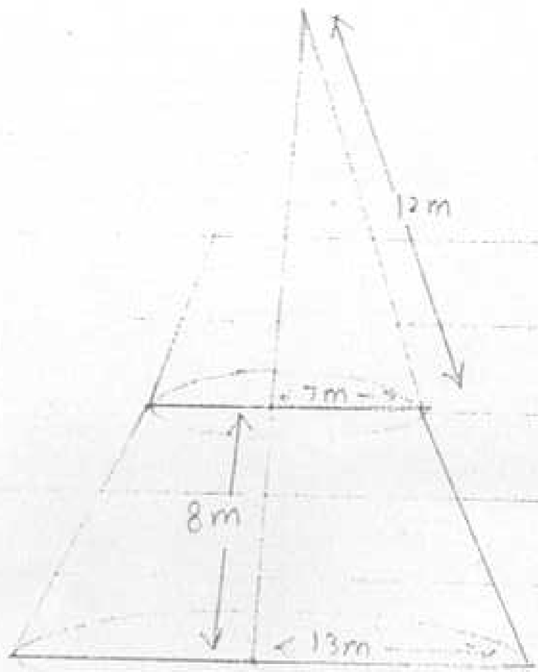
$\therefore$ The jet is flying at a constant height of $\boxed{1558.80 \text{ m}}$ or $\boxed{1.5588 \text{ km}}$.

$36 \times 18 = 648$

$1732 \times 9 = 1558.800$

$1732 \times 9 = 1558.800$

$\boxed{A\cdot 29}$



For frustum,

diameter$_1$ = d_1 = 26 m

radius$_1$ = r_1 = $\frac{d_1}{2}$ = 13 m.

diameter$_2$ = d_2 = 14 m

radius$_2$ = r_2 = $\frac{d_2}{2}$ = 7 m.

height = h = 8 m

slant height $= l = \sqrt{h^2 + (r_1 - r_2)^2}$

$= \sqrt{8^2 + (13-7)^2} \quad = \sqrt{8^2 + 6^2}$

$= \sqrt{64 + 36}$

$= \sqrt{100}$

$= 10$ m.

For cone,

diameter $= d_2 =$ 14 m.

radius $= r_2 =$ 7 m.

slant height $= l_1 =$ 12m.

Area of ~~cloth~~ canvas required = Curved surface Area of frustum + Curved Surface Area of cone $= \pi(r_1 + r_2)l + \pi r_2 l_1$

$= \pi[(13+7)(10) + (7)(12)]$

$= \pi[(20)(10) + 84]$

$= \frac{22}{7} \times 284$

$= \cancel{\frac{22}{7} \times 284}$

$= \cancel{\frac{6248}{7}}$

$= 892\,/\,7\,/\,1\,/\,4 = 892.571$

$= 892.71\ m^2 = 892.57$

~~∴ Area of canvas required is 892.71 m²~~

∴ Area of canvas required is $\boxed{892.57\ m^2}$

M-50

A-30

Class Intervals	f_i	x_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$	cf
0-10	3	5	-3	-9	3
10-20	4	15	-2	-8	7
20-30	7	25	-1	-7	14
30-40	15	35=a	0	0	29
40-50	10	45	1	10	39
50-60	7	55	2	14	46
60-70.	4.	65.	3	12	50.
	$\sum_{i=1}^{n} f_i = 50$			$\sum_{i=1}^{n} f_i u_i = 12$.	

Let assumed mean be a = 35.

Width of class intervals = h = 10.

$$\bar{x} = a + \frac{\sum_{i=1}^{n} f_i u_i}{\sum_{i=1}^{n} f_i} \times h$$

$$= 35 + \frac{12}{5\cdot} \times 10$$

$= 35 + 2.4$

$= 37.4.$

$\therefore$ Mean $= 37.4.$

No. of observations $= n = 50$ $\quad \frac{n}{2} = 25.$

Median class $= 30 - 40.$

Lower limit of median class $= l = 30.$

Cumulative frequency of class preceding median class $= 14 = cf$

Frequency of median class $= 15. = f$

Width of class intervals $= h = 10.$

$$\text{Median} = l + \frac{\frac{n}{2} - cf}{f} \times h$$

$$= 30 + \frac{25 - 14}{\cancel{15}_3} \times \cancel{10}^2 = 30 + \frac{11 \times 2}{3} = 30 + \frac{22}{3}$$

$$= 30 + 7.33 = 37.33$$

$\therefore$ Median $= 37.33$.

Modal class $= 30-40$.

Lower limit of modal class $= l = 30$.

Frequency of modal class $= f_1 = 15$.

Frequency of class preceding modal class $= f_0 = 7$.

Frequency of class succeeding modal class $= f_2 = 10$.

Width of class intervals $= h = 10$.

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 30 + \frac{15 - 7}{30 - 7 - 10} \times 10$$

$$= 30 + \frac{80}{13}$$

$= 36.15$

$\therefore$ Mode $= 36.15$.

$\therefore$ Mean $= 37.4$

Median $= 37.33$

Mode $= 36.15$.