CS-60

BACHELOR IN COMPUTER APPLICATIONS

Term-End Examination

June, 2006

CS-60 : FOUNDATION COURSE IN MATHEMATICS IN COMPUTING

Time : 3 hours *Maximum Marks : 75*

Note : *Question no. 1 is **compulsory**. Attempt any **three** questions from Q. no. 2 to 5. Calculators are not allowed.*

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1. (a) If f and g are functions defined on [a, b] such that f+g is continuous on [a, b], then must f and g be continuous ? Give reasons for your answer. 2

(b) Check whether the function $f(x) = x^2$, $\forall x \in \mathbf{R}$, is periodic or not. 2

(c) If $y = e^{m \sin^{-1} x}$, then prove that

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (m^2 + n^2)y_n = 0$$ 4

(d) Taking four sub-divisions of the interval [1, 3], find the approximate value of

$$\int_1^3 x^2 \, dx$$

using Simpson's Rule. *3*

(e) Find the condition for the line $y = mx + c$ to be a tangent to the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

4

(f) Evaluate

$$\int \frac{x^2 - 1}{x^4 + x^2 + 1} \, dx$$

5

(g) Prove that

$\cos 4\theta = \cos^4 \theta - 6 \cos^2 \theta \sin^2 \theta + \sin^4 \theta$ *3*

(h) Prove that $2^n > 1 + n\sqrt{2^{n-1}} \ \forall \ n > 2.$ *4*

(i) Find the projection of the line segment AB on the line CD, where we have A(— 2, 3, 0), B(1, — 3, 1), C(0, 0, 1), D(3, 1, 5). *3*

2. (a) Use the Mean Value Theorem, to prove that $| \sin (a - b) \cos (a + b) | \leq | a - b |$, where $a, b \in \mathbf{R}$. *4*

(b) The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ meets the axes at A, B and C. Find the equation of the cone whose vertex is the origin and whose guiding curve is the circle ABC. 5

(c) In a single Venn diagram show the sets A^c, $A \cup B$ and C, where

$A = \{x \mid x \text{ is a multiple of } 3, |x| < 20, x \in \mathbf{Z}\}$

$B = \{x \in \mathbf{N} \mid x \text{ is a factor of } 27\}$

$C = \{-\frac{1}{2}, 0, 1\}$ 3

(d) Find $\lim_{x \to \infty} \left[\sqrt{4x^2 - 3x + 5} - \sqrt{4x^2 + 4x - 5} \right]$ 3

3. (a) Show that

$$\int_0^{\pi/4} \ln(1 + \tan\theta)\, d\theta = \frac{\pi}{8} \ln 2.$$ 5

(b) Find the area enclosed by the cardioid

$$r = a(1 + \cos\theta)$$ 4

(c) Obtain the resolvent cubic, corresponding to Descartes' method, of the biquadratic equation $x^4 - 8x^3 - 6x - 2 = 0.$ 6

4. (a) Use Tchebychev's inequality to prove that

$$\sqrt{1} + \sqrt{2} + \sqrt{3} + \dots + \sqrt{n} \le n\sqrt{\frac{n+1}{2}}, \ \forall\, n \ge 1$$ 3

(b) Differentiate

$$(\sin x)^x + x^{\sin x}$$

with respect to x. 5

(c) Find the standard equation of the conicoid

$$x^2 + y^2 + z^2 - 2x - 2y - 2z - 1 = 0$$

by shifting the origin to the centre of the conicoid. 4

(d) Find all the asymptotes of the curve $x^2y = 2 + y$. 3

5. (a) Find the angle of intersection of the curves $y^2 = 3x$ and $4y^2 - x^2 = 36$. 4

(b) Find the equation of the sphere through the circle

$$x^2 + y^2 + z^2 - 4x + 2y - 6z - 22 = 0,$$
$$x + 3y - 2z = 1$$

and passing through the point $(1, 1, -1)$. 3

(c) Reduce the equation $3x^2 - 10xy + 3y^2 + 14x - 2y + 3 = 0$ to standard form. Hence identify the conic it represents. Also trace the original conic, and its reduced form. 8