MATHEMATICS - 2007

Q. 1. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of this progression equals

a. $\frac{1}{2}\left(1-\sqrt{5}\right)$
b. $\frac{1}{2}\sqrt{5}$
c. $\sqrt{5}$
d. $\frac{1}{2}\left(\sqrt{5}-1\right)$

Sol: Let geometric progression is a, ar, ar2,…………;(a, r > 0)

$$\because a = ar + ar^2 \Rightarrow r^2 + r - 1 = 0 \Rightarrow r = \frac{-1 \pm \sqrt{5}}{2} \Rightarrow r = \frac{\sqrt{5}-1}{2}$$

Correct choice: (d)

Q. 2. If sin $-1\left(\frac{x}{5}\right) + \operatorname{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$ then a value of x is.

a. 1
b. 3
c. 4
d. 5

Sol: $\sin -1\left(\frac{x}{5}\right) + \operatorname{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2} \Rightarrow \sin^{-1}\left(\frac{x}{5}\right) + \cos^{-1}\left(\frac{3}{5}\right) = \frac{\pi}{2} \Rightarrow x = 3$

Correct choice: (b)

Q. 3. In the binomial expansion of

$(a-b)^n, n \geq 5,$ *the sum of* 5^{th} *and* 6^{th} *terms in zero, then* $\frac{a}{b}$ *equals*

a. $\frac{5}{n-4}$

b. $\frac{6}{n-5}$

c. $\frac{n-5}{6}$

d. $\frac{n-4}{6}$

Sol:

$\because T_5 + T_6 = 0 \Rightarrow {}^nc_4\, a^{n-4} (-b)^4 + {}^nc_5\, a^{n-5} (-b)^5 = 0 \Rightarrow \frac{a^{n-4}\, b^4}{a^{n-5}\, b^5} = \frac{{}^nC_5}{{}^nC_4} \Rightarrow \frac{a}{b} = \frac{n-4}{5}$

Correct choice: (d)

Q. 4. The set S : = {1, 2, 3, ……, 12} is to be partitioned into three sets A, B, C of equal size. Thus, $A \cup B \cup C = S, A \cap B = B \cap C = A \cap C = \phi.$ The number of ways to partition S is

a. $\frac{12!}{3!(4!)^3}$

b. $\frac{12!}{3!(3!)^4}$

c. $\frac{12!}{(4!)^3}$

d. $\frac{12!}{(3!)^4}$

Sol: $\because$ 12 different objects are to be divided into 3 groups of equal size, which are named as set A, B and C. $\Rightarrow Number\ of\ ways = \frac{12!}{(4!)^3 . 3!} \times 3! = \frac{(12)!}{(4!)^3}$ Correct choice: (3)

Q. 5. The largest interval lying in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ for which the function

$\left[f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)\right] is\ defined, is$

a. $[0, \pi]$

b. $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
c. $\left[-\frac{\pi}{4}, \frac{\pi}{2}\right)$
d. $\left[0, \frac{\pi}{2}\right)$

a.

Sol:

$$f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)$$

4^{-x^2} *is defined for* $\forall\ x \in R$..........*(i)*

$\cos^{-1}\left(\frac{x}{2} - 1\right)$ *is defined when* $-1 \le \frac{x}{2} - 1 \le 1$, *i.e when* $0 \le x \le 4$.........*(ii)*

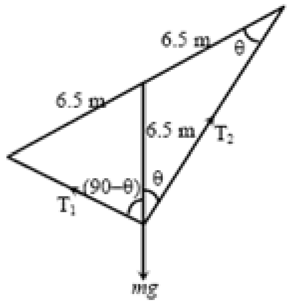
$\log\ \cos x$ *is defined when* $\cos x > 0$, *i.e when* $x \in \left(-\frac{x}{2}, \frac{x}{2}\right)$..........*(iii)*

from (i), (ii) *and* (iii), *wehave domain of* $f(x)$ *as* $\left[0, \frac{\pi}{2}\right)$

Correct choice: (d)

Q. 6. A body weighing 13 kg is suspended by two strings 5 m and 12 m long, their other ends being fastened to the extremities of a rod 13 m long. If the rod be so held that the body hangs immediately below the middle point. The tensions in the strings are

a. 12 kg and 13 kg
b. 5 kg and 5 kg
c. 5kg and 12 kg
d. 5 kg and 13 kg



$$T_2 \cos\theta + T_1 \sin\theta = mg; \quad T_2 \sin\theta + T_1 \cos\theta\, T_2 = mg \cos\theta$$

$$T_1 = mg \sin\theta; \tan\theta = \frac{5}{12}$$

Sol: $T_1 = (13\ kg)\frac{5}{13} = 5\ kg; T_2 = (13\ kg)\frac{12}{13} = 12\ kg$

So tension in the strings are 5 kg and 12 kg. Correct choice: (3)

Q. 7. A pair of fair dice is thrown independently three times. The probability of getting a score of exactly 9 twice is

a. $\frac{1}{729}$

b. $\frac{8}{9}$

c. $\frac{8}{729}$

d. $\frac{8}{243}$

Sol: Probability of getting sum of nine in a single throw $= \frac{1}{9}$

Probability of getting sum nine exactly two times out of three draws

$= {}^3C_2 \left(\frac{1}{9}\right)^2 \left(\frac{8}{9}\right) = \frac{8}{243}$.

Correct choice: (d)

Q. 8. Consider a family of circles which are passing through the point (- 1, 1) and are tangent to x-axis. If (h, k), are the co-ordinates of the centre of the circles, then the set of values of k is given by the interval

a. $0 < k < \frac{1}{2}$

b. $k \geq \frac{1}{2}$

c. $-\frac{1}{2} \leq k \leq \frac{1}{2}$

d. $\leq \frac{1}{2}$

Sol: $\because$ Centre of circle is (h, k) and x-axis is tangent. $\Rightarrow$ Radius of the family of circle $= |k|$

Also circle passes through

$$(-1, 1) \Rightarrow (1+h)^2 + (1-k)^2 = k^2 \Rightarrow h^2 + 2h + 2 - 2k = 0 \text{ for real } h, k \geq \frac{1}{2}$$

Correct choice: (b)

Q. 9. Let L be the line of intersection of the planes 2x + 3y +z = 1 and x + 3y +2z = 2. If L makes an angle α with the positive x-axis, then $\cos\alpha$ equals

a. $\frac{1}{\sqrt{3}}$

b. $\frac{1}{2}$

c. 1

d. $\frac{1}{\sqrt{2}}$

Sol:

$Vector\ normal\ to\ the\ plane\ 2x+3y+z=1\ is\ \vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$

$Vector\ normal\ to\ the\ plane\ x+3y+2z=2\ is\ \vec{b} = \hat{i} + 3\hat{j} + 2\hat{k}$

$Vector\ parallel\ to\ line\ of\ intersection\ of\ given\ planes = \vec{a} \times \vec{b} = 3\hat{i} - 3\hat{j} + 3\hat{k}$

$Angle\ \alpha\ between\ 3\hat{i} - 3\hat{j} + 3\hat{k}\ and\ \hat{i}\ is\ given\ by\ \cos\alpha = \dfrac{1}{\sqrt{3}}$

Correct choice: (a)

Q. 10. The differential equation of all circles passing through the origin and having their centres on the x-axis is

a. $x^2 = y^2 + xy\dfrac{dy}{dx}$
b. $x^2 = y^2 + 3xy\dfrac{dy}{dx}$
c. $y^2 = x^2 + 2xy\dfrac{dy}{dx}$
d. $y^2 = x^2 - 2xy\dfrac{dy}{dx}$

Sol: Equation of family of circles passing through origin and having their centres on x-axis is

$x^2 = y^2 - 2xh = 0......(i)$

$Differentiating\ w.r.t\ x\ we\ have\ 2x + 2y\dfrac{dy}{dx} - 2h = 0......(ii)$

$Eliminating\ 'h'\ from\ (i)\ and\ (ii),\ we\ have\ y^2 = x^2 + 2xy\dfrac{dy}{dx}$

Correct choice: (c)

Q. 11. If p and q are positive real numbers such that then $p^2 + q^2 = 1,$ the maximum value of (p + q) is

a. 2
b. $1/2$
c. $1/\sqrt{2}$
d. $\sqrt{2}$

Sol: $Let\ p = \cos\theta\ and\ q = \sin\theta,\ \theta \in \left(0, \frac{\pi}{2}\right) \Rightarrow p + q = \cos\theta + \sin\theta \le \sqrt{2}$

Correct choice: (d)

Q. 12. A tower stands at the centre of a circular park. A and B are two points on the boundary of the park such that $AB\ (= a)$ subtends an angle of 60^0 at the foot of the tower, and the angle of elevation of the top of the tower from A or B is 30^0. The height of the tower is

a. $\frac{2a}{\sqrt{3}}$

b. $2a\sqrt{3}$

c. $\frac{a}{\sqrt{3}}$

d. $a\sqrt{3}$

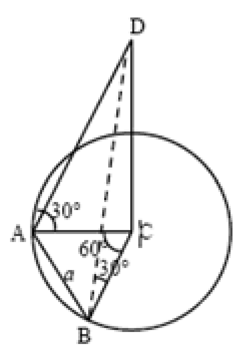
Sol:

$\because \angle ACB = 60^0$

$\Rightarrow Triangle\ ABC\ is\ an\ equilateral\ triangle$

$\Rightarrow Radius\ of\ circle = a$

$Now\ \frac{DC}{AC} = \tan 30^0 \Rightarrow DC = \frac{a}{\sqrt{3}}$



Correct choice: (c)

Q. 13. The sum of the series $^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \ldots\ldots - \ldots. + {}^{20}C_{10}\ is$

a. $-\ {}^{20}C_{10}$
b. $\frac{1}{2}\ {}^{20}C_{10}$
c. 0
d. ${}^{20}C_{10}$

Sol:

$$\because\ {}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \ldots\ -\ {}^{20}C_9 + {}^{20}C_{10} - {}^{20}C_{11} + {}^{20}C_{12} - \ldots\ldots + {}^{20}C_{20} = 0$$
$$\Rightarrow 2\left\{{}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - \ldots\ldots - {}^{20}C_9\right\} + {}^{20}C_{10} = 0$$
$$\Rightarrow 2\left\{{}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - \ldots\ldots - {}^{20}C_9 + {}^{20}C_{10}\right\} = {}^{20}C_{10}$$
$$\Rightarrow {}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - \ldots\ldots + {}^{20}C_{10} = \frac{1}{2}\ {}^{20}C_{10}$$

Correct choice: (b)

Q. 14. The normal to a curve at P(x, y) meets the x-axis at G. If the distance of G from the origin is twice the abscissa of P, then the curve is a

a. ellipse
b. parabola
c. circle
d. hyperbola

Sol:

$$\because\ |x + my| = 2x \Rightarrow x + my = \pm 2x \Rightarrow ydy = xdx \text{ or } ydy$$
$$= -3xdx \Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + (\text{which is hyperbola})$$
$$\text{or } \frac{3x^2}{2} + \frac{y^2}{2} = k\ (\text{which is an ellipse})$$

Correct choice: (a, d)

Q. 15. $\text{If } |z+4| \le 3, \text{ then the maximum value of } |z+1| \text{ is}$

a. 4
b. 10
c. 6

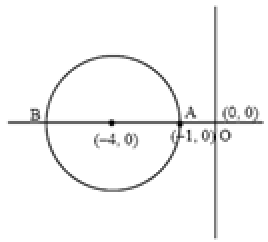
d. 0

Sol:

$If\ |z+4| \le 3, then\ the\ \max imum\ value\ of\ |z+1|\ is$

$\Rightarrow z\ lies\ inside\ or\ on\ the\ circle\ of\ radius\ 3\ and\ centre\ at\ (-4, 0)$

$\Rightarrow \max imum\ value\ of\ |z+1|\ is\ 6.$



Correct choice: (c)

Q. 16. The resultant of two forces P N and 3 N is a force of 7 N. If the direction of 3 N force were reversed, the resultant would be $\sqrt{19}\ N$. The value of P is

a. 5 N
b. 6 N
c. 3 N
d. 4 N

Sol:

$\because 7^2 = P^2 + 9 + 6P\cos\theta \Rightarrow 6P\cos\theta = 40 - P^2$

$and\ 19 = P^2 + 9 + 6P\cos(\pi - \theta) \Rightarrow 19 = P^2 + 9 - 6P\cos\theta$

$\Rightarrow 19 = P^2 + 9 - 40 + P^2\ \{u\sin g\ (i)\}$

$\Rightarrow P = 5\ N$

Correct choice: (a)

Q. 17. Two aeroplanes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2, respectively. The second plane will bomb only if the first misses the target. The probability that the target is hit by the second plane is

a. 0.06

b. 0.14
c. 0.2
d. 0.7

Sol:

$$P(I) = 0.3.\ P(II) = 0.2$$

$$\text{Required probability} = P(\bar{I}).\ P(II)\,P(\bar{I})\,P(II) + \ldots\ldots$$

$$= (0.7)(0.2) + (0.7)(0.8)(0.7)(0.2) + \ldots\ldots$$

$$= \frac{7}{22}$$

No choice is correct

Q. 18. $If\ D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}\ for\ x \neq 0, y \neq 0\ then\ D\ is$

a. divisible by neither x nor y
b. divisible by both x and y
c. divisible by x but not y
d. divisible by y but not x

Sol:

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix} C_3 \to C_3 - C_1$$

$$C_2 \to C_2 - C_1$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 1 & x & 0 \\ 1 & 0 & y \end{vmatrix}$$

Correct choice: (b)

Q. 19. For the hyperbola $\frac{x^2}{\cos^2 \alpha} - \frac{y^2}{\sin^2 \alpha} = 1,$ which of the following remains constant when α varies?

a. Eccentricity
b. Directrix
c. Abscissae of vertices
d. Abscissae of foci

Sol:

$$\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1,$$
$$\because b^2 = a^2\left(e^2 - 1\right) \Rightarrow \sin^2\alpha = \cos^2\alpha\left(e^2 - 1\right) \Rightarrow e^2 = \tan^2\alpha + 1 = \sec^2\alpha$$
$$\Rightarrow e = \sec\alpha$$
$$Directrix : x = \pm\frac{a}{e} = \pm\cos^2\alpha$$
$$Abscissae\ of\ vertices = \pm a = \pm\cos\alpha$$
$$ABCISSAE\ F\ FOCI = \pm ae = \pm\cos\alpha . \sec\alpha = \pm 1$$

Correct choice: (d)

Q. 20. If a line makes an angle of $\frac{\pi}{4}$ with the positive directions of each of x-axis and y-axis, then the angle that the line makes with the positive direction of the z-axis is

a. $\frac{\pi}{6}$
b. $\frac{\pi}{3}$
c. $\frac{\pi}{4}$
d. $\frac{\pi}{2}$

Sol:

$$\because \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \quad \Rightarrow \frac{1}{2} + \frac{1}{2} + \cos^2\gamma = 1\ \ as\ \alpha = \beta = \frac{\pi}{4}$$
$$\Rightarrow \cos^2\gamma = 0 \Rightarrow \gamma = \frac{\pi}{2}$$

Correct choice: (d)

Q. 21. A value of C for which the conclusion of Mean Value Theorem holds for the function $f(x) = \log_e x$ on the interval [1, 3] is

a. $2\log_3 e$
b. $\frac{1}{2}\log_e 3$
c. $\log_3 e$
d. $\log_e 3$

Sol: $f'(C) = \frac{f(3) - f(1)}{2} \Rightarrow \frac{1}{C} = \frac{\log_e 3}{2} \Rightarrow C = 2\log_3 e$

Correct choice: (a)

Q. 22. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in

a. $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$
b. $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$
c. $\left(0, \frac{\pi}{2}\right)$
d. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Sol:

$f(x) = \tan^{-1}(\sin x + \cos x) is\ increa\sin g\ if\ (\sin x + \cos x)\ is\ increa\sin g$

$\Rightarrow \cos x - \sin x > 0 \quad \Rightarrow \quad \cos x > \sin x \quad \Rightarrow \quad x \in \left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$

Correct choice: (b)

Q. 23. $Let\ A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}. If\ \left|A^2\right| = 25, then \left|\alpha\right|\ equals$

a. 5^2
b. 1
c. $\frac{1}{5}$
d. 5

Sol:

$$A=\begin{bmatrix}5 & 5\alpha & \alpha\\ 0 & \alpha & 5\alpha\\ 0 & 0 & 5\end{bmatrix}\Rightarrow |A.A|=|A|\,|A|=(25\,\alpha)^2=25\Rightarrow \alpha^2=\frac{1}{25}\Rightarrow \alpha=\pm\frac{1}{5}$$

Correct choice: (c)

Q. 24. $The\ sum\ of\ series\ \frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\ldots..\ upto\ \inf inity\ is$

a. e^{-2}
b. e^{-1}
c. $e^{-\frac{1}{2}}$
d. $e^{+\frac{1}{2}}$

Sol:

$$e^x=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots\ldots\quad Put\ x=-1$$

$$\Rightarrow\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\ldots\ldots\ldots=e^{-1}$$

Correct choice: (b)

Q. 25. If $\hat{u}\ and\ \hat{v}$ are unit vectors and θ is the acute angle between them, then $2\,\hat{u}\times 3\,\hat{v}$ is a unit vector for

a. Exactly two values of θ
b. More than two values of θ
c. No value of θ

d. Exactly one value of θ

Sol:

$$|2\hat{u} \times 3\hat{v}| 6 |\hat{u} \times \hat{v}| = 1$$

$$\Rightarrow |\hat{u} \times \hat{v}| = \frac{1}{6} \Rightarrow \sin \theta = \pm \frac{1}{6}$$

$$As\ \theta\ is\ acute\ angle\ for\ \sin \theta = \frac{1}{6}. So\ \theta\ can\ take\ only\ one\ Value.$$

Correct choice: (d)

Q. 26. A particle just clears a wall of height b at a distance a and strikes the ground at a distance c from the point of projection. The angle of projection is

a. $\tan^{-1} \frac{b}{ac}$
b. 45^0
c. $\tan^{-1} \frac{bc}{a(c-a)}$
d. $\tan^{-1} \frac{bc}{a}$

Sol:

$$b = a \tan \alpha - \frac{1}{2} \frac{ga^2}{u^2 \cos^2 \alpha}\ (equation\ of\ trajectory)$$

$$So, c = \frac{u^2 \sin 2\alpha}{g} \Rightarrow \frac{g}{u^2} = \frac{\sin 2\alpha}{c}$$

$$So, b = a \tan \alpha - \frac{1}{2} \times \frac{a^2 \sin 2\alpha}{c \cos^2 \alpha} \Rightarrow b = a \tan \alpha - \frac{a^2}{c} \tan \alpha$$

$$\Rightarrow b = \left(\frac{ac - a^2}{c}\right) \tan \alpha \Rightarrow \tan \alpha = \frac{bc}{a(c-a)}$$

Correct choice: (c)

Q. 27. The average marks of boys in a class is 52 and that of girls is 42. The average marks of boys and girls combined is 50. The percentage of boys in the class is

a. 40
b. 20
c. 80
d. 60

Sol: Let there are x boys and y girls in the class. So, total marks are
52 x + 42 y and according to second condition total marks = (x + y) 50
Now, $52x + 42y = 50x + 50y \Rightarrow 2x = 8y \Rightarrow x = 4$
So percentage of boys is 80.

Correct choice: (c)

Q. 28. The equation of a tangent to the parabola $y^2 = 8x$ is $y = x + 2$. The point on this line from which the other tangent to the parabola is perpendicular to the given tangent is

a. (- 1, 1)
b. (0, 2)
c. (2, 4)
d. (- 2, 0)

Sol: Point of intersection of two perpendicular tangents to the parabola lies on directrix of the parabola. Equation of directrix is x + 2 = 0
So point is (- 2, 0)

Correct choice: (d)

Q.29. If (2, 3, 5) is one end of a diameter of the sphere $x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$, then the coordinates of the other end of the diameter are

a. (4, 9, - 3)
b. (4, - 3, 3)
c. (4, 3, 5)
d. (4, 3, - 3)

Sol: Centre (3, 6, 1)
Let other end is (α, β, γ)

$$So, \frac{\alpha + 2}{2} = 3, \frac{\beta + 3}{2} = 6, \frac{\gamma + 5}{2} = 1 \Rightarrow \alpha = 4, \beta = 9, \gamma = -3$$

Correct choice: (a)

Q. 30. $Let\ \bar{a} = \hat{i} + \hat{j} + \hat{k}, \bar{b} = \hat{i} - \hat{j} + 2\hat{k}\ and\ \bar{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}.$
If the vector $\bar{c}$ *lies in the plane of* $\bar{a}$ *and* $\bar{b}$, *then* x *equals*

a. 0
b. 1
c. – 4
d. – 2

Sol:

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ x & x-2 & -1 \end{vmatrix} = 0 \Rightarrow 1(1 - 2(x-2)) - 1(-1-2x) + 1(x - 2 + x) = 0$$
$$\Rightarrow 2x + 4 = 0 \Rightarrow x = -2$$

Correct choice: (d)

Q. 31. Let A (h, k), B (1, 1) and C (2, 1) be the vertices of a right angled triangle with AC as its hypotenuse. If the area of the triangle is 1, then the set of values which 'k' can take is given by

a. $\{1, 3\}$
b. $\{0, 2\}$
c. $\{-1, 3\}$
d. $\{-3, -2\}$

Sol: $A = \frac{1}{2} . 1. |k-1| = 1 \Rightarrow k - 1 = 2\ or\ -2 \Rightarrow k = 3, -1$

Correct choice: (c)

Q. 32. $Let\ P = (-1, 0), Q = (0, 0)\ and\ R = (3, 3\sqrt{3})$ be three points. The equation of the bisector of the angle PQR is

a. $\sqrt{3}x + y = 0$
b. $x + \frac{\sqrt{3}}{2}y = 0$

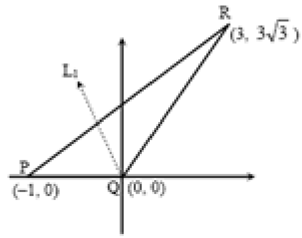
c. $\frac{\sqrt{3}}{2}x + y = 0$
d. $x + \sqrt{3}y = 0$

Sol:

$Slope\ of\ QR = \sqrt{3}\ .\ So.\ \angle PQR = 120^0$

$So\ m_{L1} = \tan 120^0 = -\sqrt{3}$

$So\ equation\ of\ L_1\ is\ y = -\sqrt{3}x$



Correct choice: (a)

Q. 33. If one of the lines of $my^2 + (1-m^2)xy - mx^2 = 0$ is a bisector of the angle between the lines xy = 0, then m is

a. $-\frac{1}{2}$
b. – 2
c. 1
d. 2

Sol:

$my^2 + (1-m^2)xy - mx^2 = 0 \Rightarrow (y - mx)(my + x)0 \Rightarrow y = mx, -\frac{1}{m}x$

$So\ m = 1\ or\ -1$

Correct choice: (c)

Q. 34. $Let\ F(x) = f(x) + f\left(\frac{1}{x}\right), where\ f(x) = \int_1^x \frac{\log t}{1+t}dt\ .\ Then\ F(e)\ equals$

a. $\frac{1}{2}$
b. 0
c. 1
d. 2

Sol:

$$F(x) = \int_1^x \frac{\log t}{1+t}\,dt + \int_1^{\frac{1}{x}} \frac{\log t}{1+t}\,dt = \int_1^x \frac{\log t}{1+t}\,dt + \int_1^x \frac{\log t}{t.(1+t)}\,dt$$

$$= \int_1^x \frac{\log t}{1+t}\,dt \; \frac{(\log_e x)^2}{2}. \; So\ F(e) = \frac{1}{2}$$

Correct choice: (a)

Q. 35. $Let\ f : R \to R\ be\ a\ function\ defined\ by\ f(x) = Min\ \{x+1, |x|+1\}$. Then which of the following is true?

a. $f(x) \geq 1\ for\ all\ x \in R$
b. f (x) is not differentiable at x = 1.
c. f (x) is differentiable everywhere
d. f (x) is not differentiable at x = 0.

Sol: $f(x) = x + 1,\ \forall\ x \in R$

Correct choice: (c)

Q. 36. $The\ function\ f : R \setminus \{0\} \to R\ given\ by\ f(x) = \frac{1}{x} - \frac{2}{e^{2x} - 1}$ can be made continuous at x = 0 by defining f (0) as

a. 2
b. – 1
c. 0
d. 1

Sol:

$$f\ (0) = \lim_{x \to 0} \frac{e^{2x} - 1\ 2x}{x\left(e^{2x} - 1\right)} = 2 \lim_{1 \to 0} \frac{e^{t} - 1 -}{t\ .\left(e^{t} - 1\right)}$$

$$= 2 \lim_{1 \to 0} \frac{e^{t} - 1}{te^{t} + e^{t} - 1} \qquad \textit{By L'Hospital Rule}$$

$$= 2 \lim_{1 \to 0} \frac{e^{t} - 1}{te^{t} + e^{t} + e^{t}} \qquad \textit{Again by L'Hospital Rule}$$

$$= 2 \times \frac{1}{2} = 1$$

Correct choice: (d)

Q. 37. $\int \frac{dx}{\cos x + \sqrt{3} \sin x}$ *equals*

a. $\frac{1}{2} \log \tan \left(\frac{x}{2} + \frac{\pi}{12}\right) + C$

b. $\frac{1}{2} \log \tan \left(\frac{x}{2} - \frac{\pi}{12}\right) + C$

c. $\frac{1}{2} \log \tan \left(\frac{x}{2} - \frac{\pi}{12}\right) + C$

d. $\log \tan \left(\frac{x}{2} + \frac{\pi}{12}\right) + C$

Sol:

$$\int \frac{dx}{\cos x + \sqrt{3} \sin x} \frac{1}{2} \int \frac{dx}{\frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \cos x} = \frac{1}{2} \int \frac{dx}{\sin \left(\frac{\pi}{6} + x\right)}$$

$$\Rightarrow \frac{1}{4} \int \frac{dx}{\sin \left(\frac{\pi}{12} + \frac{x}{2}\right) \cos \left(\frac{\pi}{12} + \frac{x}{2}\right)} = \frac{1}{2} \log \tan \left(\frac{\pi}{12} + \frac{x}{2}\right) + C$$

Correct choice: (a)

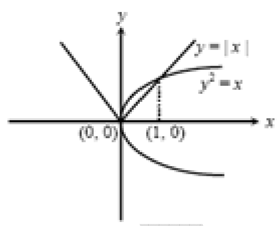
Q. 38. The area enclosed between the curves $y^2 = x \textit{ and } y = |x| \textit{is}$

a. $\frac{2}{3}$
b. 1
c. $\frac{1}{6}$
d. $\frac{1}{3}$

Sol:

$y^2 = x \text{ and } y = |x| \Rightarrow x^2 = x \Rightarrow x = 0 \text{ or } 1$

$$\text{Required area} = \int_0^1 \left(\sqrt{x} - x\right)dx = \left[\frac{2}{3}x^{3/2} - \frac{x^2}{2}\right]_0^1 = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$$



Correct choice: (c)

Q. 39. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$ then the set of possible values of a is

a. (- 3, 3)
b. $(-3, \infty)$
c. $(3, \infty)$
d. $-\infty, -3$

Sol:

$\alpha + \beta = -a$

$|\alpha - \beta| < \sqrt{5} \Rightarrow (\alpha - \beta)^2 < 5 \Rightarrow a^2 - 4 < 5 \Rightarrow a \in (-3, 3)$

Correct choice: (a)